

Dear students,
Good morning .
Study materials for today (30th April,
2020) :

1. Topic 8.5 Equations of Motion by
graphical method and the topic

8.5.1 Equation for Velocity-time
relation .

2. Go through the video clips and do the
following :

1. Write all the 3 equations of motion
with the descriptions of the u , v , a , t and
 s .

2. Practice two times the 1st
equation of motion with graphical
representation .

Home work will be uploaded by 6 pm
(Friday, 01st May) .

That's all for today .

Activity 8.10

- Feroz and his sister Sania go to school on their bicycles. Both of them start at the same time from their home but take different times to reach the school although they follow the same route. Table 8.5 shows the distance travelled by them in different times

Table 8.5: Distance covered by Feroz and Sania at different times on their bicycles

Time	Distance travelled by Feroz (km)	Distance travelled by Sania (km)
8:00 am	0	0
8:05 am	1.0	0.8
8:10 am	1.9	1.6
8:15 am	2.8	2.3
8:20 am	3.6	3.0
8:25 am	–	3.6

- Plot the distance-time graph for their motions on the same scale and interpret.



Questions

- What is the nature of the distance-time graphs for uniform and non-uniform motion of an object?
- What can you say about the motion of an object whose distance-time graph is a straight line parallel to the time axis?
- What can you say about the motion of an object if its speed-time graph is a straight line parallel to the time axis?
- What is the quantity which is measured by the area occupied below the velocity-time graph?

8.5 Equations of Motion by Graphical Method

When an object moves along a straight line with uniform acceleration, it is possible to relate its velocity, acceleration during motion and the distance covered by it in a certain time interval by a set of equations known as the equations of motion. There are three such equations. These are:

$$v = u + at \quad (8.5)$$

$$s = ut + \frac{1}{2} at^2 \quad (8.6)$$

$$2as = v^2 - u^2 \quad (8.7)$$

where u is the initial velocity of the object which moves with uniform acceleration a for time t , v is the final velocity, and s is the distance travelled by the object in time t . Eq. (8.5) describes the velocity-time relation and Eq. (8.6) represents the position-time relation. Eq. (8.7), which represents the relation between the position and the velocity, can be obtained from Eqs. (8.5) and (8.6) by eliminating t . These three equations can be derived by graphical method.

8.5.1 EQUATION FOR VELOCITY-TIME RELATION

Consider the velocity-time graph of an object that moves under uniform acceleration as

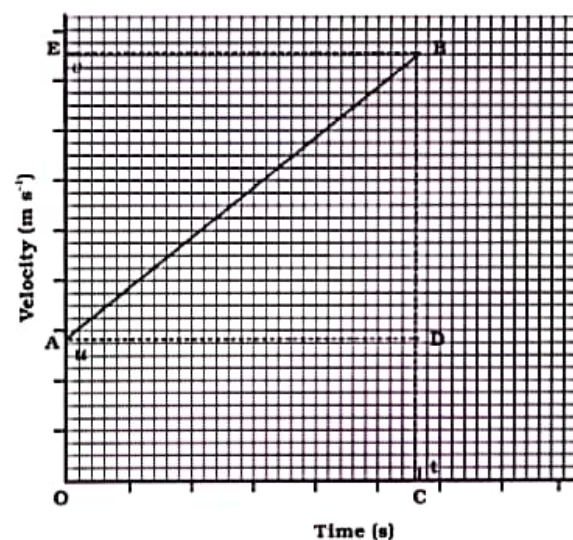


Fig. 8.8: Velocity-time graph to obtain the equations of motion

shown in Fig. 8.8 (similar to Fig. 8.6, but now with $u \neq 0$). From this graph, you can see that initial velocity of the object is u (at point A) and then it increases to v (at point B) in time t . The velocity changes at a uniform rate a . In Fig. 8.8, the perpendicular lines BC and BE are drawn from point B on the time and the velocity axes respectively, so that the initial velocity is represented by OA, the final velocity is represented by BC and the time interval t is represented by OC. $BD = BC - CD$, represents the change in velocity in time interval t .

Let us draw AD parallel to OC. From the graph, we observe that

$$\begin{aligned} BC &= BD + DC = BD + OA \\ \text{Substituting } BC &= v \text{ and } OA = u, \\ \text{we get } v &= BD + u \\ \text{or } BD &= v - u \end{aligned} \quad (8.8)$$

From the velocity-time graph (Fig. 8.8), the acceleration of the object is given by

$$\begin{aligned} a &= \frac{\text{Change in velocity}}{\text{time taken}} \\ &= \frac{BD}{AD} = \frac{BD}{OC} \end{aligned}$$

Substituting $OC = t$, we get

$$\begin{aligned} a &= \frac{BD}{t} \\ \text{or } BD &= at \end{aligned} \quad (8.9)$$

Using Eqs. (8.8) and (8.9) we get

$$v = u + at$$

8.5.2 EQUATION FOR POSITION-TIME RELATION

Let us consider that the object has travelled a distance s in time t under uniform acceleration a . In Fig. 8.8, the distance travelled by the object is obtained by the area enclosed within OABC under the velocity-time graph AB.

Thus, the distance s travelled by the object is given by

$$\begin{aligned} s &= \text{area OABC (which is a trapezium)} \\ &= \text{area of the rectangle OADC} + \text{area of the triangle ABD} \end{aligned}$$

$$= OA \times OC + \frac{1}{2} (AD \times BD) \quad (8.10)$$

Substituting $OA = u$, $OC = AD = t$ and $BD = at$, we get

$$\begin{aligned} s &= u \times t + \frac{1}{2} (t \times at) \\ \text{or } s &= ut + \frac{1}{2} at^2 \end{aligned}$$

8.5.3 EQUATION FOR POSITION-VELOCITY RELATION

From the velocity-time graph shown in Fig. 8.8, the distance s travelled by the object in time t , moving under uniform acceleration a is given by the area enclosed within the trapezium OABC under the graph. That is,

$s = \text{area of the trapezium OABC}$

$$= \frac{(OA + BC) \times OC}{2}$$

Substituting $OA = u$, $BC = v$ and $OC = t$, we get

$$s = \frac{(u + v)t}{2} \quad (8.11)$$

From the velocity-time relation (Eq. 8.6), we get

$$t = \frac{(v - u)}{a} \quad (8.12)$$

Using Eqs. (8.11) and (8.12) we have

$$s = \frac{(v + u) \times (v - u)}{2a}$$

$$\text{or } 2as = v^2 - u^2$$

Example 8.5 A train starting from rest attains a velocity of 72 km h^{-1} in 5 minutes. Assuming that the acceleration is uniform, find (i) the acceleration and (ii) the distance travelled by the train for attaining this velocity.